Non-extensive distributions for a relativistic Fermi Gas

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Abstract: Recently the non-extensive approach has been used in a variety of ways to describe dense nuclear matter. They differ in the methods of introducing the appropriate non-extensive single particle distributions inside a relativistic many-body system, in particular when one has to deal both with particles and antiparticles, as in the case of quark matter exemplified in the NJL approach. I present and discuss in detail the physical consequences of the methods used so far, which should be recognized before any physical conclusions can be reached from the results presented.

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It is nowadays widely accepted that in many branches of physics experimental data indicate the necessity of a departure from the standard extensive Boltzman-Gibs statistics, which is then replaced by a non-extensive statistics [1]. In high energy scattering one uses Tsallis statistics and Tsallis distributions which describe particle distributions over a large energy region with the help of only one single additional parameter, the non-extensivity $q \neq 1$. This replaces all additional parameters necessary when the extensive description is used; for $q = 1$ one recovers the usual BG statistics (cf. [2] and references therein). When considering dense Nuclear Matter (NM) some kind of mean field theory is used, either in terms of nuclear degrees of freedom (like the Walecka model (WM) [3]), or in terms of quark and anti-quark degrees of freedom (like in the Nambu-Jona-Lasino (NJL) model [4,5,6]). Both descriptions has been reformulated using a non-extensive approach (for the q-WM in [7,8,9,10] and for the q-NJL model in [11]) *. All these models try to find the non-extensive traces in the nuclear Equation of State (EOS) using the non-extensive single particle distributions. However, it is very difficult to compare their results in a conclusive way because they use different forms of single, non-extensive particle distribu-

* Other approaches to non-extensive dense matter can be found in [12,13,14].

tions. In this work, analyzing a Fermi Gas model we compare and discuss these distributions following the method presented in [15], both for the momentum dependent fermion distributions $n_q(p)$ and for anti-fermion distributions $\bar{n}_q(\bar{p})$.

In the extensive approach to dense, hot matter the particle and antiparticle occupation numbers, n_i and $\bar{n}_i$, can be obtained from the Jayne's extremalization of the entropic measure

$$S = \sum_i \left[n_i \ln n_i + (1 - n_i) \ln(1 - n_i) \right] + \left[n_i \to \bar{n}_i \right], \tag{1}$$

under the constraints imposed by the total number of particles, N, and the total energy of the system, E (ϵ_i is the energy of the i-th energy level) [16],

$$\sum_i (n_i - \bar{n}_i) = N \qquad \text{and} \qquad \sum_i (n_i + \bar{n}_i)\, \epsilon_i = E. \tag{2}$$

As a result we get the Fermi-Dirac distributions,

$$n_i = \frac{1}{\exp(x_i) + 1}, \qquad \bar{n}_i = \frac{1}{\exp(\bar{x}_i) + 1}, \tag{3}$$

which depend on the dimensionless quantities

$$x = \beta(\epsilon - \mu) \qquad \text{and} \qquad \bar{x} = \beta(\epsilon + \mu). \tag{4}$$

where $\beta = 1/T$, $\epsilon = \sqrt{p^2 + m^2}$, m is fermion mass and μ its chemical potential. For $\epsilon = 0$, the distributions $n(x)$ and $\bar{n}(\bar{x}) = n(\bar{x})$ satisfy following relation:

$$n(x) + n(\bar{x}) = 1. \tag{5}$$

The chemical potential (separation energy) for anti-fermions is negative $-\mu$ (because the N increases after removing the anti-fermion), therefore this relation allows holes among the negative energy states to be interpreted as antiparticles with positive energy. As we shall see below, this is usually not the case for non-extensive distributions [17]. The similar relation for particle distributions

$$n(x) + n(-x) = 1 \tag{6}$$

exhibits, particle-hole symmetry around the Fermi surface where $x = 0$.

When formulating a non-extensive version of the model of dense matter one must do it in a thermodynamically consistent way. This means that one has to preserve the standard thermodynamical relationships among the thermodynamical variables such as, for example, entropy, energy and temperature,

$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}. \tag{7}$$

The best solution is to use once more Jayne's maximum entropy prescription and extremalize the appropriate non-extensive entropic measures with specifically chosen constraints because, as was shown in [18], any thermostatistical formalism constructed by this method complies with the thermodynamical relationships [15]. A thermodynamically consistent formulation is then obtained by the appropriate identification of relevant constraints with extensive thermodynamical quantities (like number of particles N or energy E) and of the corresponding Lagrange multipliers with appropriate intensive thermodynamical quantities (like temperature T and chemical potential μ). In order to describe non-extensive many-body systems characterized by $q \neq 1$, one has to introduce the non-extensive measure of the entropy expressed, in analogy with Eq. (1), in terms of single particle distributions for fermions and anti-fermions, with accordingly modified constraints (2). In this way we obtain a q-generalized quantum distributions from a given form of the non-extensive entropy.

There are different choices of non-extensive entropy which lead to different thermodynamical solutions [19,15,20]. We shall discuss three possibilities, two of which are currently exploited [9,13]. In the first choice, one uses a straightforward q-generalization of the entropic measure used in Eq.(1), which now has the following form:

$$S_q = \sum_i s_{qi} \qquad \text{where for} \quad 0 < n_{qi}(\bar{n}_{qi}) < 1$$

$$(\bullet) \qquad s_{qi} = \frac{1 - n_{qi}^q - (1 - n_{qi})^q}{q-1} + \{n_{qi} \to \bar{n}_{qi}\}\,. \tag{8}$$

This generalization consists in replacing in Eq.(1) $\ln(n)$ by $\ln_q(n_q) = (1 - n_q^{1-q})/(1-q)$ and in using as the corresponding effective occupation numbers of particles and antiparticles, n_{qi}^q and $\bar{n}_{qi}^q$, instead of n_i and $\bar{n}_i$. This reflects the fact that non-extensive properties can be formally understood as a presence of

some effective interaction between the constituents of the system, the strength of which is proportional to $|q-1|$ (in the non-extensive environment, parameterized by q, constituents are no longer free particles) [21]. Therefore constraints (2) for the total energy E of the system and the conservation of the total fermion number N will now have the following form **:

$$\sum_i \left(n_{qi}^q - \bar{n}_{qi}^q\right) = N \qquad \text{and} \qquad \sum_i \left(n_{qi}^q + \bar{n}_{qi}^q\right)\epsilon_i = E. \tag{9}$$

The extremalization of entropy given by Eq.(8), performed under constraints (9), with respect to two independent variables, n and $\bar{n}$, gives the following set of equations with Lagrange multipliers α and β:

$$\frac{\partial s_q}{\partial n_q}=(\alpha+\beta\epsilon)qn_q^{(q-1)}; \quad \frac{\partial s_q}{\partial \bar{n}_q} = (-\alpha+\beta\epsilon)q\bar{n}_q^{(q-1)} \ \text{ for } \ 0<n_q(\bar{n}_q)<1, \tag{10}$$

Solving these one obtains the following single particle distributions for fermions and anti-fermions:

$$n_q=\frac{1}{e_q(x)+1}, \qquad \bar{n}_q = \frac{1}{e_q(\bar{x})+1} \qquad \text{with} \quad e_q(x) = [1+(q-1)x]^{\frac{1}{q-1}}, \tag{11}$$

where x and $\bar{x}$ are the same as in Eq.(4) with chemical potential $\mu = -\alpha/\beta$. However, these solutions must be supplemented by conditions assuring that $\mathrm{e}_q(x)$ in Eq.(11) is properly defined, i.e. that

$$1+(q-1)x \geq 0$$

for all q and $x(\bar{x})$. We get therefore four constraints for particle momenta p and antiparticle momenta $\bar{p}$ with $q<1$ and $q>1$:

$$\sqrt{p^2+m^2}\leq -\frac{T}{q-1}+\mu, \qquad \sqrt{\bar{p}^2+m^2} \leq -\frac{T}{q-1}-\mu \quad \text{for} \quad q\leq 1; \tag{12}$$

$$\sqrt{p^2+m^2}\geq -\frac{T}{q-1}+\mu, \qquad \sqrt{\bar{p}^2+m^2} \geq -\frac{T}{q-1}-\mu \quad \text{for} \quad q> 1. \tag{13}$$

These lead to the so-called Tsallis' cut-off prescription for single particle distributions:

** This form assures the fulfillment of basic requirements of thermodynamical consistency [19,15,20].

$$n_{q<1} = 0 \quad \text{for} \quad \epsilon > \mu + \frac{\beta}{(1-q)}, \quad \bar{n}_{q<1} = 0 \quad \text{for} \quad \epsilon > -\mu + \frac{\beta}{(1-q)} \tag{14}$$

$$n_{q>1} = 1 \quad \text{for} \quad \epsilon < \mu - \frac{\beta}{(q-1)}, \qquad \text{no limitations for} \quad \bar{n}_{q>1}. \tag{15}$$

Notice differences in restrictions imposed on momenta p and $\bar{p}$ for $q > 1$ and $q < 1$ in Eqs. (12) and (13). For $q > 1$ and sufficiently small momenta p, the limiting energy, $\epsilon_{lim} \leq -T/(q-1) + \mu$, gives the usual prescription (15) used in [9]. For $q < 1$ and for large momenta p, the limiting energy, $\epsilon_{lim} > -T/(q-1)+\mu$, cuts off large momenta, well above the Fermi level. However, limits given in (12) for anti-fermion momenta practically prevent their propagation (as particle-antiparticle pair excitations) for small temperatures, see Eq.(14). Consequently, this choice of S_q allow only for $q > 1$ in the propagation of virtual particle-antiparticle pairs. This means that for S_q given by Eq.(8) only the $q > 1$ case has physical meaning in relativistic Fermi gas model. Note that relations (5) and (6) are not now satisfied because $e_q(x) * e_q(-x) \neq 1$.

In order to allow for a relativistic calculation with $q < 1$ and including virtual particle-antiparticle pairs, we propose to use the entropy functional (cf. 8) but with a different (dual) parameter q for antiparticles,

$$q \rightarrow \hat{q} = 2 - q.$$

This is the second choice to be discussed. In this case we have:

$$\left(\begin{matrix}\bullet \\ \bullet\end{matrix}\right) \quad s_{qi} = \frac{1 - n_{qi}^{q} - (1 - n_{qi})^{q}}{q - 1} + \{n_{qi} \rightarrow \bar{n}_{qi}, q \rightarrow \hat{q}\} \quad \text{for} \;\; 0 < n_{qi}(\hat{n}_{qi}) < 1 \tag{16}$$

As a result, after extremalisation, with the same conditions as before, one obtains the following occupation numbers:

$$n_q = \frac{1}{e_q(x) + 1}, \qquad \bar{n}_{\hat{q}} = \frac{1}{\hat{e}_q(\bar{x}) + 1}, \quad \text{with} \quad \hat{e}_q(x) = [1 + (\hat{q} - 1)]^{\frac{1}{\hat{q}-1}}. \tag{17}$$

Note, that now the corresponding effective occupation numbers are n_q^q for particles (as before) and $\bar{n}_q^{\hat{q}}$ for antiparticles. The non-extensive analog of relation (5) is now satisfied because dual $\hat{q} = 2 - q$ in the anti-fermion distribution in Eq.(17) means that $\hat{e}_q(-x) = 1/e_q(x)$. Therefore

$$n_q(x) + \bar{n}_{\hat{q}}(-x) = 1. \tag{18}$$

To summarize, the $q > 1$ relativistic dynamics will be realized in the first choice of S_q, given by Eq.(8), whereas $q < 1$ dynamics will be realized in the second choice of S_q, Eq.(16). However, only in the later case is the property (18) satisfied.

The third choice of non-extensive entropy consists in removing all previous additional cut-off's by using a new definition of entropy, S_q (cf. 8). It has been proposed already in [15] but without including anti-fermions. In our case this choice means that:
(*) we assume $q > 1$ for fermions above the Fermi sea,
(*) we assume $q < 1$ for fermions in the Fermi sea,
(*) we assume $q > 1$ for anti-fermions.
The resulting s_{qi} has the following form for $q > 1$ (cf. 8):

$$\begin{pmatrix} \bullet \\ \bullet \\ \bullet \end{pmatrix} \quad s_{qi} = \begin{cases} \frac{1-n_{qi}^{q}-(1-n_{qi})^{q}}{q-1} + \{n_{qi} \to \bar{n}_{qi}\} & \text{for} \quad 0 \le n_{qi} \le 1/2 \\ \\ \frac{1-n_{qi}^{\hat{q}}-(1-n_{qi})^{\hat{q}}}{\hat{q}-1} + \{n_{qi} \to \bar{n}_{qi}, \hat{q} \to q\} & \text{for} \quad 1/2 < n_{qi} \le 1 \end{cases} \tag{19}$$

Extremalizing S_q with the same conditions as given by Eq.(9) one obtains:

$$\begin{aligned} n_q &= \begin{cases} \frac{1}{1+e_q(\beta(\epsilon-\mu))} & \text{for} \;\; 0 \le n_q \le 1/2, \\ \frac{1}{1+e_{\hat{q}}(\beta(\epsilon-\mu))} & \text{for} \;\; 1/2 < n_q \le 1, \end{cases} \\ \bar{n}_q &= \frac{1}{1+e_q(\beta(\epsilon+\mu))} \qquad\qquad \text{for} \quad 0 \le \bar{n}_q \le 1. \end{aligned} \tag{20}$$

In this case the corresponding effective occupation numbers are $n_q^{\hat{q}}$ for particles in the Fermi sea, n_q^q for particles above the Fermi level and $\bar{n}_q^q$ for antiparticles.

Because the q parameter is now different (dual) for particles below the Fermi surface and particles above the Fermi surface, the following non-extensive version of relation (6) (obtained similarly to (18)), which connects particle occupations around the Fermi surface is satisfied:

$$n_q(x) + n_{\hat{q}}(-x) = 1 \ . \tag{21}$$

Note that in a non-extensive Fermi gas one deals with effective occupations, $n_q^{eff} = (n_q)^q$, as was mentioned before. Therefore, relations (18) and (21) for the single particle distributions n_q and $\bar{n}_q$ will not be satisfied for the effective distributions n_q^{eff} and $\bar{n}_q^{eff}$. $\star\star\star$

To summarize, in this work we derive the fermion $n_q(p)$ and anti-fermion $\bar{n}_q(\bar{p})$ distributions in the Fermi gas model for 3 different choices of non-extensive entropy, which correspond to different choices of non-extensive parameterizations [15]. These choices result in different descriptions of dense NM [8,9,11,13,14]. In the first two choices, given in Eqs.(8) and (16), the non-extensivity parameter q is constant for all values of $n_q(\bar{n}_q)$. Limits imposed by Eq.(12) on the anti-fermion distribution allows its propagation only for $q > 1$. Therefore, in the first choice the particle and anti-particle distributions can be obtained for the same $q > 1$ [9] but without particle-antiparticle symmetry (18). This symmetry is restored in the second choice, where we choose an anti-fermion distribution with dual $\hat{q} = 2 - q$ for anti-fermions for all values of $n_q(\bar{n}_q)$. The third choice, Eq.(19), introduces a jump in the q parameter at $x = 0$ for particles which results in discontinuous behavior in the single particle entropy, single particle energy and effective occupation, n^{eff}, which in the non-extensive case has the power q [13].

The first choice, Eq.(8), realized for $q > 1$, with a cut off (15) for small momenta p, introduces in general positive correlations which increase the particle occupation numbers at the bottom of the Fermi sea. The second choice, Eq.(16), with cut-off (15) for large momenta p in the tail of the Fermi distribution, introduces correlations which eliminate large momenta well above the Fermi sea. However, this scenario is not supported by recent experiments which exhibit strong short range correlations with a large tail in the distribution above the Fermi sea [22]. The third choice, Eq.(19), involves a change of the parameter q in the particle distribution at the Fermi surface. This change of parameter q from ($\hat{q} = 2 - q < 1$) to ($q > 1$) means that we expect different, non-extensive effects below and above the Fermi surface. However, the jump in the q value at the Fermi surface produces an energy gap near the Fermi surface. Whereas such

$\star\star\star$However, the departures from unity are small because $q \to \hat{q} = 2 - q$ in the power of the effective distribution n_q^{eff}.

a gap is well known in nuclear physics and is produced by strong pairing correlations in the nuclear ground state below the Fermi surface, it disappears when the temperature increases [23]. Therefore, the above non-extensive description, which assumes a change of the q parameter at the Fermi surface, acts in the opposite direction and will not describe this phenomenon. On the other hand, if we modify the change in q in such a way that it goes smoothly from $\bar{q}$ in the Fermi sea to q above the Fermi sea, within the finite range of momenta near the Fermi surface, then the occupation and energy gaps can vanish. In this way we will get the restoration of standard Boltzman-Gibs statistics, with $q = 1$, at the Fermi surface.

Such jumps in q value are absent for antiparticles. One has to stress that their role is vital in NJL models. In the WM type of approach they are usually neglected because in these models the effective degrees of freedom are projected onto the positive energy states (nucleons). However, the negative energy states (anti-nucleons) cannot be neglected in NM [3,24,25]. In the relativistic mean field model there are possible corrections coming from the vacuum polarization which involve contributions from nucleon-antinucleon pairs [3,25]. Therefore, in the non-extensive version of these models (like in qWM) [7,8,11], one has to decide whether higher order $N\bar{N}$ loops should be included and for $q > 1$ (first choice) or ($q < 1$) (second choice). The nuclear EOS described in quark and anti-quark degrees of freedom is discussed in the q-NJL model [11] approach. Here the propagation of quark-antiquark pairs, described by $q > 1$ statistics, is essential for the dynamics of massless quarks which change their phase at the critical point. The non-extensive corrections spread the critical point over a wider area of temperature and density [11].

A description of the EOS in terms of nuclear degrees of freedom is well formulated in WM [3]. There are two groups which generalize this mean field approach. The constant non-extensive parameter $q > 1$ is applied in [10]. The others [8,11,14] apply different distributions which cannot be obtained directly from extremalization but they satisfy basic thermodynamical relations[14]. Their n_q distributions are formally identical to those obtained from our third choice (and also to those obtained in [20]) but later they use [8,11,14] a constant power q in $n_q^{eff} = n_q^q$. Only in [13] the q power changes in the Fermi surface, like in our results for the third choice given in Eq.(19). Therefore, only the results of [13] are fully consistent.

The following relation for the effective distribution n_q^{eff} shows its behavior around the Fermi surface - where $n_q(x=0)=1/2$:

$$\begin{array}{llll} n_q^q(x)+n_q^q(-x)<1 & \text{for} & q>1 & \text{1st choice Eq.(8)}, \\ n_q^q(x)+n_q^q(-x)>1 & \text{for} & q<1 & \text{2nd choice Eq.(16)}, \\ n_q^q(x)+n_q^q(-x)\simeq 1 & \text{for} & q\approx 1 & \text{3rd choice Eq.(19)}. \end{array} \tag{22}$$

Here we assume for the third choice (19) that the jump in parameter q is smeared for particle momenta around $x=0$ with $q=1$ at the Fermi surface. These properties should now be compared with realistic distribution of large nucleon momenta induced by short range correlations observed recently in [22]. Such a comparison should allow for a better understanding of the role of the parameter q.

Concluding, different q parameterizations presented in this work predict different non-extensive effects in different part of the particle spectra. The main difference was found at the Fermi surface between smooth non-extensive dynamics with constant q, (with $q>1$ for the first choice and $q<1$ for the second one), against the behavior observed in the third choice (19), characterized by a substantial change in the non-extensive parameter q across the $q=1$ value on the Fermi surface. Observations which will indicate the restoration of standard Boltzman-Gibs statistics ($q=1$) at the Fermi surface would support the third (19) choice which is used [13] in the calculation of the EOS with nucleon and meson degrees of freedom. The relations (23) which reflect the broken symmetries of extensive dynamics, should be accounted for in any non-extensive analysis in NM. It is interesting to note that changes in the non-extensive parameter q (like assumed in (19)) can produce energy gaps, which are in general connected with attractive correlations in Fermi systems.

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